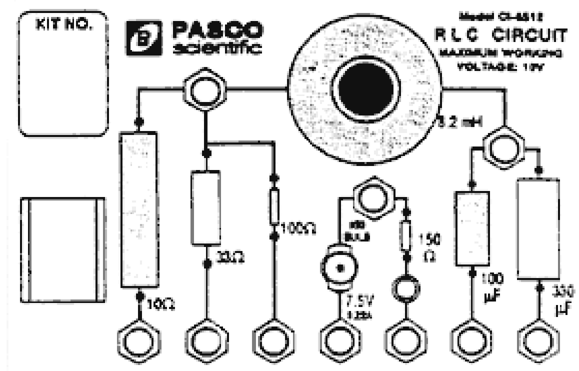
RCL Circuits[1]

Equipment Needed:

2 RCL Circuit Experiment Board	Demo Cart
1 Voltage Sensor Assembly	Demo Cart
1 Knife Switch (Double Throw)	Demo Cart
1 RCL meter	Demo Desk
1 Computer Cart/Interface and Power Amp	

Figure 1: Component Board



Purpose

The purpose of this experiment is to study resonance in an series RCL (L - inductance, R - resistance, C - capacitance) circuit by examining the response (current) of the circuit as a function of the frequency of the applied voltage.

Theory

The amplitude of the AC current (I_0) in a series RCL circuit depends on the amplitude of the applied voltage (V_0) and the impedance (Z).

$$I_0 = \frac{V_0}{Z},$$

where Z is the impedance.

Since the impedance depends on frequency, the current varies with frequency:

$$Z = \sqrt{R^2 + (X_L - X_C)^2},$$

[1]Adapted from PASCO ©1994, PASCO Scientific Roseville, CA

where $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$ are the reactance, and $\omega = 2\pi f$.

The current will be maximum when the circuit is driven at its resonant frequency:

$$\omega_{resonance} = \frac{1}{\sqrt{LC}}.$$

If the resonant frequency is plugged into the equation for the impedance (Z), one can show that $X_L - X_C$ at resonance and thus the impedance is equal to R. Therefore, at resonance the impedance is the lowest value possible and the current will be the largest possible.

Figures 2 through 4 show the voltage applied to the same RCL circuit and the resulting current for three different frequencies. Figure 2 resulted from applying a voltage at a frequency lower than the resonant frequency. The voltage and current are out of phase (the peaks do not coincide). The current leads the voltage for $\omega < \omega_{\mathbf{res}}$.

Figure 2: RCL circuit below resonance, $\omega < \omega_{resonance}$.

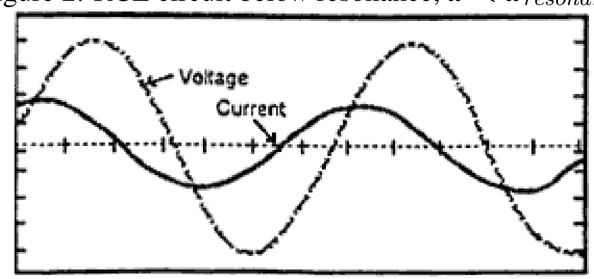


Figure 3 shows the result of applying a voltage at the resonant frequency of the circuit. The voltage and current are in phase and the current has the largest amplitude at resonance.

Figure 3: RCL circuit at resonance, $\omega = \omega_{resonance}$.

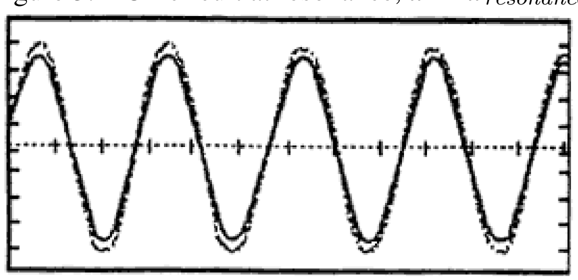
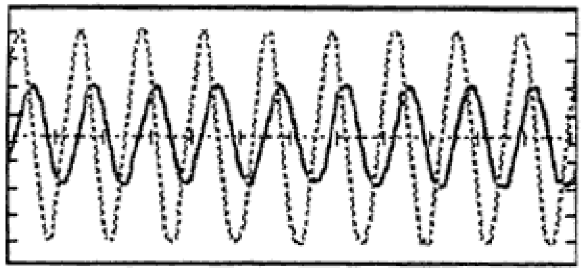


Figure 4 shows the result of applying a voltage at a frequency higher than the resonant frequency. Again the voltage and current are out of phase and the current is less than it is at resonance. For $\omega > \omega_{\mathbf{res}}$, the current lags behind the voltage.

In this experiment the amplitude of the current vs. frequency is plotted. Since the current is a maximum at the resonant frequency and is less for greater or lesser frequencies, the graph is expected to peak at the resonant frequency.

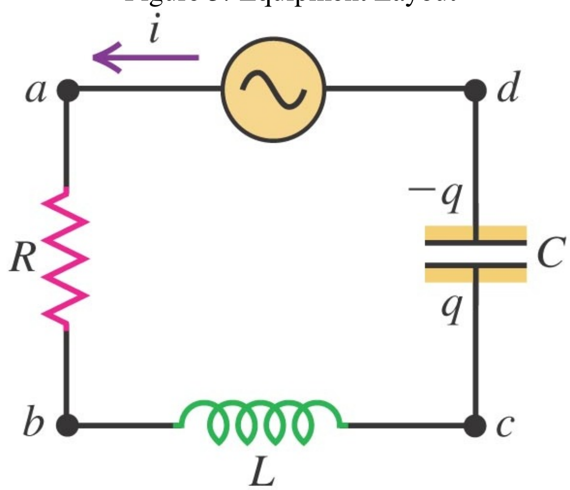
Figure 4: RCL circuit above resonance, $\omega > \omega_{resonance}$.



Setup

Construct the circuit shown in Figure 5 using the output from the power amplifier as the voltage source. With the Voltage Sensor DIN plugged into Channel A, connect the leads as shown. Insert the core into the center of the coil.

Figure 5: Equipment Layout



Procedure

1. Set up the circuit in Figure 5 with $R = 10\Omega$, $C = 100\mu F$ and $L = 8mH$ (without core).
2. The values above are only approximate, so you should get the precise value by direct measurement. Measure the resistance of the resistor and the inductor, the inductance and the capacitance using the RCL meter.
3. Turn on the interface box and computer. Choose the program "Data Studio" on the computer. Click on "Create Experiment" after the program begins.
4. The `Experiment Setup` window automatically opens. Click on the image of channel A. From the popup list scroll all the way down to select `Voltage Sensor` and click `OK`.

5. Click on the power output (the rightmost terminal on the image) and a small popup window named `Signal Generator` should open. We will use the default `Sine Wave`. Change the output frequency to $100Hz$. You may keep the default amplitude of $5V$. Leave the `Signal Generator` window open when you are done.
6. Connect channel A across the resistor. The voltage across the resistor is denoted V_R below, and we are primarily interested in the amplitude of this voltage. By Ohm's law, $V_R = IR$, so measuring V_R is essentially the same as measuring I.
7. Double click on `Scope` and select `Output Voltage`.
8. Double click on `Scope` again and select `Voltage, ChA` for the resistor voltage measured by channel A.
9. Under the `Displays` panel on the left, under `Scope`, triple click on `Scope 1` and `Scope 2` to change the names to `Output Voltage` and `Resistor Voltage`.
10. Click `Start` and you should see the sinusoidal curves on the graphs. At this point you will see the curves are highly unstable, it jumps left and right continuously. The way to get a stable curve is to select the rising trigger (second button from the left, looks like a white triangle pointing up). Do it for both graphs.
11. Play around with the frequency and observe that the peak resistor voltage changes with the frequency (no need to take any data yet, just checking to make sure it is working properly).
12. Increase the frequency at roughly $50Hz$ increment from $1Hz$ to $500Hz$ (i.e. $1Hz, 50Hz, 100Hz, \cdots$). Record the amplitude of V_R in Table 1.
13. Figure out the approximate frequency at which V_R peaks and collect more data points in the vicinity of the peak at roughly $10Hz$ intervals.
14. Repeat the same measurement again this time with the core in the inductor and record your results in Table 2.
15. Repeat the measurement with the big inductor. This time you want to use the approximate inductance of $L \approx 830mH$ and $f_{resonance} = \frac{1}{2\pi\sqrt{LC}}$ to estimate where the peak is, and do the frequency sweep centered around this resonant frequency. Record your results in Table 3. When you connect the big inductor, you cannot simply connect it in parallel with the small coil (otherwise the small coil will dominate and you will not see the effect of the big coil). The way to do this is to use a wire to by-pass the small inductor, and let the current passes from R to C and only then feed the current into the big inductor and back into the power supply.
16. Plot V_R versus ω (not f!) for Table 1, 2 and 3 in three separate graphs.

Data

Measurements using RCL meter:

$R_{resistor} =$ ________________

Table 1: The frequency dependence of the amplitude of V_R without core in inductor

f (Hz)											
ω (rad/s)											
V_R amplitude (V)											
	Continue Below										
f (Hz)											
ω (rad/s)											
V_R amplitude (V)											

$V_{R,resonance} =$ ________________

$f_{resonance} =$ ________________

$\omega_{resonance} =$ ________________

Table 2: The frequency dependence of the amplitude of V_R with core in inductor

f (Hz)											
ω (rad/s)											
V_R amplitude (V)											
	Continue Below										
f (Hz)											
ω (rad/s)											
V_R amplitude (V)											

$V_{R,resonance} =$ ________________

$f_{resonance} =$ ________________

$\omega_{resonance} =$ ________________

Table 3: The frequency dependence of the amplitude of V_R with big inductor

f (Hz)											
ω (rad/s)											
V_R amplitude (V)											
	Continue Below										
f (Hz)											
ω (rad/s)											
V_R amplitude (V)											

$V_{R,resonance} =$ ________________

$f_{resonance} =$ ________________

$\omega_{resonance} =$ ________________

$R_{inductor} =$ ________________

$L_{meter}(no\ core) =$ ________________

$L_{meter}(with\ core) =$ ________________

$C_{meter} =$ ________________

Data Analysis

1. Using the resonant frequency found from the screen, calculate the resonant angular frequency using:

$$\omega_{theory} = \frac{1}{\sqrt{L_{meter}C_{meter}}}.$$

   Use the values of L, C from your measurements from the RCL meter.

2. Compare the measured resonant angular frequency $\omega_{resonance}$ from Table 1 and 2 to the theoretical value ω_{theory} for both cases (with core and without).

3. Using $\omega_{resonance}$, we can calculate L using:

$$L_{computer} = \frac{1}{\omega^2_{resonance}C_{meter}}.$$

See if this value matches with the your measurements from the RCL meter.

Table 4: Comparison

	Without core	With core	Big Coil
ω_{theory} (rad/s)			
$\omega_{resonance}$ (rad/s)			
Percentage difference of ω			
$L_{computer}$ (mH)			
L_{meter} (mH)			
Percentage difference of L			